

## **MATHEMATICS** EXCELLENCE KEY AGYAT GUPTA (M.Sc., M.Phil.)



पजियन क्रमांक

**REGNO:-TMC-D/79/89/36** 

## Pre-Board Examination 2011 -12

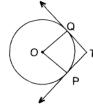
Time:  $3 \text{ to } 3 \frac{1}{4}$  Hours अधिकतम समय : 3 से 3 🏒

Maximum Marks: 80 अधिकतम अंक : 80 Total No. Of Pages: 4 कुल पृष्ठों की संख्या: 4

CLASS - X CBSE (SA-2)**MATHEMATICS** 

### **SECTION A**

- **Q.1** If one root of the equation  $ax^2 + bx + c = 0$  is three times the other, then
- (a)  $2b^2 = 9ac$  (b)  $b^2 = 16ac$  (c)  $b^2 = ac$  (d)  $3b^2 = 16ac$  . Ans d All Aces, Jacks and Queens are removed from a deck of playing cards. One card is **Q.2** drawn at random from the remaining cards. then the probability that the card
  - drawn is not a face card. (A) 1/10 (B) 1/9 (C)  $\frac{9}{10}$  (D) none
- **Q.3** Two tangents TP and TQ are drawn from an external point T to a circle with centre at O, as shown in Fig. 2. If they are inclined to each other at an angle of 100° then



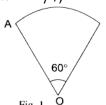
what is the value of  $\angle POQ$ ?

Fig. 2

(A)  $60^{\circ}$  (B)  $110^{\circ}$  (C)  $100^{\circ}$  (D)  $80^{\circ}$ 

Ans d

- If the numbers a, b, c, d, e form an AP, then the value of a-4b+6c-4d+e is **Q.4**
- (a) 1 (b) 2 (c) 0 (d) none of these Ans: c
- What is the distance between two parallel tangents of a circle of radius 4 cm? **Q.5** (A) 12 cm (B) 4 cm (C) 8 cm (D) none
- If Figure is a sector of a circle of radius 10.5 cm, find the perimeter of the sector. **Q.6** (Take  $\pi = \frac{22}{7}$ )



(A) 32 cm (B) 11 cm (C) 66 cm (D) none Ans a

- **Q.7** If  $\alpha, \beta$  are roots of the equation  $x^2 + 5x + 5 = 0$ , then equation whose roots are  $\alpha + 1$  and  $\beta + 1$  is
  - (a)  $x^2 + 5x 5 = 0$  (b)  $x^2 + 3x + 5 = 0$  (c)  $x^2 + 3x + 1 = 0$  (d) none of these Ans.c

- The length of the tangent from a point A at a distance of 5 cm from the centre of the 0.8 circle is 4 cm. What will be the radius of the circle?
  - (A) 3 cm (B) 4 cm (C) 3 m (D) none Ans a
- The radii of the circular bases of frustum of a right circular cone are 12 cm and 3 cm and height is 12 **Q.9** cm. Find the total surface area
  - (a)  $378 \pi cm^2$
- (b)  $2268 \pi cm^2$
- (c)  $378 \, cm^{-2}$
- (d) none of these Ans.a
- An electrician has to repair an electric fault on a pole of height 6 m. he needs to reach a point 2.54 m Q.10 below the top of the pole. What should be the length of ladder that he should use which when inclined at an angle of  $60^{\circ}$  to the horizontal would enable him to reach the desired point? (take  $\sqrt{3} = 1.73$ ) (a) 3.46 m (b) 4 m (c) 5.19 m (d) 7.5 m Ans.b

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### **SECTION - B**

In what ratio does the point P(2, -5) divide the line segment joining A(-3, 5) and B(4,-**Q.11** 1 (4, -9) <sub>k</sub> (2, -5) Let AP : PB = k : 1

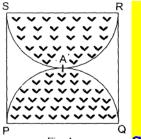
Coordinates of P = Coordinates of P

CBSE

$$\left(\frac{4k-3}{k+1}, \frac{-9k+5}{k+1}\right) = (2, -5) \dots \text{(Using Section formula)} \therefore \quad \frac{4k-3}{k+1} = \frac{2}{1} \Rightarrow 4k - 3 = 2k + 2 \Rightarrow 4k$$

 $2k = 2 + 3 \Rightarrow 2k = 5 \Rightarrow k = 5/2$ . Required Ratio = k : 1 = 5/2 : 1 = 5 : 2

PQRS is a square land of side 28 m, Two semicircular grass covered portions are to Q.12 be made on two of its opposite sides as shown in Figure 4. How much area will be



left uncovered? (Take  $\pi$ = 22/7)

Area left uncovered

Area (semicircle

= Area (square  
= 
$$(28 \times 28) \text{ m}^2 - 2\left(\frac{\pi}{2}(14)^2\right) \text{m}^2$$

$$= \left(784 - \frac{22}{7} \times 14 \times 14\right) \text{m}^2$$

$$= (784 - 616) \text{ m}^2$$

$$= 168 \text{ m}^2$$

$$\begin{bmatrix} \because \text{Ar. of Square} = (\text{side})^2 \\ \text{Ar. of Circle} = \pi r^2 \\ \text{Side} = 28 \text{ m} \\ \text{Radius} = r = \frac{28}{2} = 14 \text{ m} \end{bmatrix}$$

Prove that the point (a, 0), (0, b) and (1, 1) are collinear if  $\frac{1}{1} + \frac{1}{1} = 1$ . Q.13

Find a point on the y-axis which is equidistant from the points A(6,5) and B(-4, 3). Sol. Let (0, y) be a point on the y-axis equidistant from A (6, 5) and B (-4, 3)  $\Rightarrow$  PA =  $\sqrt{(6-0)^2 + (5-y)^2}$  Now, PA = PB  $\Rightarrow$  (PA)<sup>2</sup> = (PB)<sup>2</sup> ... (Squaring both

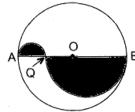
PA = 
$$\sqrt{(6-0)^2 + (5-y)^2}$$
  
=  $\sqrt{y^2 - 10y + 61}$   
PB =  $\sqrt{(-4-0)^2 + (3-y)^2}$   
=  $\sqrt{y^2 - 6y + 25}$ 

Now, PA = PB  $\Rightarrow$  (PA)<sup>2</sup> = (PB)<sup>2</sup> ...(Squaring both both production of the production

 $= \sqrt{y^2 - 6y + 25}$ sides)  $\Rightarrow y^2 - 10y + 61 = y^2 - by + 25 \Rightarrow y^2 - 10y - y^2 + 6y = 25 - 61 \Rightarrow -4y = -36 \Rightarrow y = 9$ .

Required point is (0, 9)

- A bag contains 5 red balls, 8 green balls and 7 white balls. One ball is drawn at Q.14 random from the bag. Find the probability of getting:
  - (i) a white ball or a green ball.
  - (ii) neither a green ball nor a red ball. **Sol.** Total number of balls = 5 + 8 + 7 = 20
  - (i) P (white or green ball) =  $\frac{15}{20} = \frac{3}{4}$  (ii) P (neither green nor red) =  $\frac{7}{20}$
- Find the area of the shaded region of Fig. 8, if the diameter of the circle with centre O is 28 cm and AQ Q.15



Sol. Diameter  $AQ = 1/4 \times 28 = 7 \text{cm}$ 

 $\Rightarrow r = \frac{7}{2} \text{cm} \text{ Diameter QB} = \frac{3}{4} \times 28 = 21 \text{ cm} \Rightarrow R = \frac{21}{2} \text{ cm} \text{ Area of shaded region} = \frac{1}{2} (\pi r^2 + \pi R^2)$ 

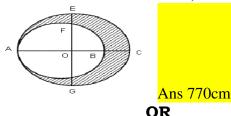
- $= \frac{\pi}{2}(r^2 + R^2) = \frac{1}{2} \cdot \pi \left[ \left( \frac{7}{2} \right)^2 + \left( \frac{21}{2} \right)^2 \right] = \frac{1}{2} \times \frac{22}{7} \times \left( \frac{49}{4} + \frac{441}{4} \right) = \frac{1}{2} \times \frac{22}{7} \times \left( \frac{49 + 441}{4} \right) = \frac{11}{7} \times \frac{490}{4} = \frac{770}{4} = 192.5 \text{ cm}^2.$ A circle touches the side BC of a  $\triangle$ ABC at a point P and touches AB and AC when produced at Q and **Q.16** R respectively. Show that:  $AQ = \frac{1}{2}$  (Perimeter of  $\triangle ABC$ ).
- Solve for x:  $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}(x \neq 2,4)$ . Ans  $x=5,\frac{5}{2}$ Q.17
- Determine an A.P. whose 3rd term is 16 and when 5th term is subtracted from the 7th Q.18 term, we get 12. **Sol.** Let the A.P. be a, a + d, a + 2d,.....a is the first term and

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*d* is the common difference. Using  $a_n = a + (n - 1) d$  **A.T.Q.** $a + 2d = 16(a_3 = 16)...(ii)$ (a + 6d)-(a + 4d)=12 $(a_7 - a_s$ = 12) ...(ii) From (ii), a + 6d-a-4d = 12 . 2d= 12  $\Rightarrow$  d = 6 Putting the value of d in (i), we get  $a=16-2d\Rightarrow$  a =16-2(6) = 4 ... Required A.P. =

### SECTION - C

0.19 In the given figure, O is the centre of the bigger circle and AC is its diameter. Another circle with AB as diameter is drawn. If AC=54 cm and BC=10 cm, Find the area of the shaded region.



The interior of a building is in the form of a right circular cylinder of radius 7 m and height 6m, surmounted by a right circular cone of same radius and of vertical angle  $60^{\circ}$ . Find the cost of painting the building from inside at the rate of Rs.  $30/\text{m}^2$ . Sol.

Internal curved surface area of cylinder =  $27 \pi rh = (2\pi \times 7 \times 6)m^2$ 

$$= \left(2 \times \frac{22}{7} \times 7 \times 6\right) \text{m}^2$$

$$= 264 \text{ m}^2 \text{ In right} \triangle \text{OAB.} \quad \frac{\text{AB}}{\text{OB}} = \sin 30^\circ$$
Slant  $\frac{7}{\text{OB}} = \frac{1}{2} \text{ height of cone (OB)} = 14 \text{ m}$ 

Internal curved surface area of cone =  $\pi$ rl =  $\frac{22}{7}$ × 7× 14 = 308m<sup>2</sup> Total area to be painted =  $(264 + 308) = 572 \text{ m}^2 \text{ Cost of painting } @ \text{Rs. } 30 \text{ per m}^2 =$ Rs.  $(30 \times 572) = Rs. 17,160$ 

The Points A(2, 9), B(a, 5), C(5, 5) are the vertices of a triangle ABC right angled at B. Find the value Q.20 of 'a' and hence the area of  $\triangle ABC$ . Ans  $\triangle$  ABC is right angled triangle; right angled at B,

BY pythagoras theorem, we get  $(AC)^2 = (AB)^2 + (BC)^2$ 

Using distance formula, we have  $\{(5-2)^2 + (5-9)^2\} = \{a-2)^2 + (5-9)^2\} + \{(5-a)^2 + (5-5)^2\}$ 

$$25 = 2a^{2} - 14a + 45$$

$$9 + 16 = a^{2} + 4 - 4a + 16 + 25 + a^{2} - 10a$$

$$2a^{2} - 14a + 20 = 0 = a^{2} - 7a + 10 = 0$$

$$a^{2} - 5a - 2a + 10 = 0$$

$$a(a - 5) - 2(a - 5) = 0 \Rightarrow (a - 2)(a - 5) = 0 \Rightarrow$$

Either a - 2 = 0 or a - 5 = 0. a = 2 or a = 5 but a cannot be 5. [if a = 5, then point B and C coincides a=2 Now  $area(\Delta ABC) = \frac{1}{2} \times AB \times BC = \frac{1}{2} \sqrt{[(2-2)^2 + (9-5)^2]} \times \sqrt{[(5-2)^2 + (5-5)^2]} = \frac{1}{2} \times 4 \times 3 = 6 \text{ sq.units}$ 

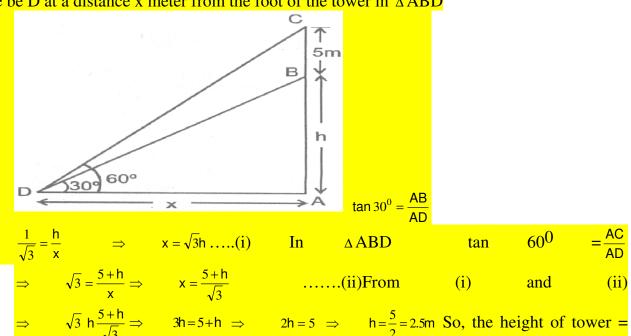
- If the 10th term of an A.P. is 47 and its first term is 2, find the sum of its first 15 **Q.21** terms. **Sol.** Let a be the first term and d be the common difference of an A.P.  $a_{10} = 47$ , a = 2 (Given), ...(i)  $\Rightarrow$  a + 9d = 47 [:  $a_n = a + (n-1)d$ ]  $\Rightarrow$ 47 = 2 +(10 - 1)d  $\Rightarrow$ 47  $= 2 + 9d \Rightarrow 9d = 47 - 2 = 45 : d = \frac{45}{9} = 5 S_n = \frac{n}{2} [2\alpha + (n-1)d] : S_{15} = \frac{15}{2} [2(2) + (15-1)(5)]$  $\Rightarrow S_{15} = \frac{15}{2} [4 + (14) (5)] \Rightarrow S_{15} = \frac{15}{2} [4 + 7C] \Rightarrow S_{15} = \frac{15}{2} [74]. \therefore S_{15} = 15 (37) = 555$
- The coordinates of the vertices of  $\triangle ABC$  are A (4,1), B (-3, 2) and C (0, k). Given that Q.22 the area of  $\triangle ABC$  is 12 units<sup>2</sup>, find the value of k. **Sol.** Ar  $(\triangle ABC) = 12$  units<sup>2</sup> (Given) [4 (2-k) + (-3) (k - 1) + 0(1 - 2)] = 12units<sup>2</sup>  $[8-4k-3k+3] = \pm 24 \cdot 11 - 7k = \pm 24 \cdot 7k = \pm 24 - 11$

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Q.28 A vertical tower stands on a horizontal plane and is surmounted by vertical flag staff of height 5 meters. At a point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30<sup>0</sup> and 60<sup>0</sup> find the height of tower. ANS :Let AB be the tower of

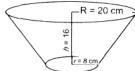
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height h metre and BC be the height of flag staff surmounted on the tower, Let the point of the place be D at a distance x meter from the foot of the tower in  $\triangle$  ABD



### **SECTION - D**

- A container (open at the top) made up of a metal sheet is in the form of a frustum of a cone of height **Q.29** 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find:
  - (i) the cost of milk when it is completely Filled with milk at the rate of Rs. 15 per litre.
  - (ii) the cost of metal sheet used, if it costs Rs. 5 per 100 cm<sup>2</sup>. (Take  $\pi = 3.14$ )Sol. The container is in the shape of a frustum of a cone . h = 16 cm, r = 8 cm, R = 20 cm



Volume 0f the container =  $\frac{1}{3} \times \pi h$  (R<sup>2</sup> + Rr + r<sup>2</sup>) =  $\frac{1}{3}$  x 3.14 x 16 [(20)<sup>2</sup> + 20(8) + (8)<sup>2</sup>] cm<sup>3</sup> =  $(\frac{1}{3} \times 3.14 \times 16 \times 624)$ cm<sup>3</sup>  $= \frac{1}{3} \times 3.14 \times 16 (400 + 160 + 64) \text{ cm}^{3} = \frac{10449.92 \text{ cm}^{3}}{1000} \text{ litres}$   $= \frac{1}{3} \times 3.14 \times 16 (400 + 160 + 64) \text{ cm}^{3} = \frac{10449.92}{1000} \text{ litres}$   $= \frac{1}{1000} \times 3.16 \times 3.16$ 

= Rs. 156.75 Now, slant height of the frustum of cone . L =  $\sqrt{h^2 + (R - r)^2} = \sqrt{16^2 + (20 - 8)^2}$  $=\sqrt{256+144}$  =  $\sqrt{400}$  = 20 cm.

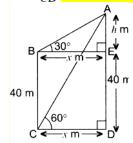
Total surface area of the container=  $[\pi l \ (R + r) + \pi r^2]$ =  $[3.14 \times 20 \ (20 + 8) + 3.14 \ (8)^2] \ cm^2$ 

= 3.14 [20x28 + 64] cm<sup>2</sup> = 3.14 x 624 cm<sup>2</sup> = 1959.36 cm<sup>2</sup> (ii) Cost of metal sheet used= Rs.  $\left[1959.36 \times \frac{5}{100}\right] = \frac{9796.8}{100}$ =Rs. 97.968= Rs. 98 (approx.)

From the top and foot of a tower 40 m high, the angle of elevation of the top of a lighthouse is found to Q.30 be 30° and 60° respectively. Find the height of the lighthouse. Also find the distance of the top of the

lighthouse from the foot of the tower. Sol. Let 
$$AE = h$$
 m and  $BE = CD = x$  m

$$\therefore \frac{x}{h} = \cot 30^{\circ} \Rightarrow \frac{x}{h} = \sqrt{3} \Rightarrow x = h\sqrt{3} \qquad ...(i) \Rightarrow BE = CD = h\sqrt{3} \text{ m}$$
In rt.  $\triangle ADC$ ,  $\frac{AD}{CD} = \tan 60^{\circ} \Rightarrow \frac{h+40}{x} = \sqrt{3} \Rightarrow h+40 = \sqrt{3} x$ 



⇒ h + 40=  $\sqrt{3} \times h \sqrt{3}$  ... [From(i)⇒40 = 3h-h ⇒ 2h = 40⇒h = 20m. Height of lighthouse = 20 + 40 = 60 m . Inrt.  $\triangle ADC$ ,  $\frac{AD}{AC} = \sin 60^{\circ}$   $\frac{60}{AC} = \frac{\sqrt{3}}{2}$  ⇒  $\sqrt{3}AC = 60 \times 2$  ⇒AC - 60 x 2/ $\sqrt{3}$ 

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# CLASS – X CBSE (SA-2) MATHEMATICS $\Rightarrow AC = 60 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow AC = \frac{60 \times 2 \times \sqrt{3}}{3} \Rightarrow AC = 40\sqrt{3} \text{m}. \text{ Hence the distance of the top of lighthouse}$ from the foot of the tower = $40\sqrt{3}$ m Prove that sum of n term of A . P . is $S_n = \frac{n}{2} [2a + (n-1)d]$ . Q.31 A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for first day, Rs. 250 for second day, Rs. 300 for third day and so on. If the contractor pays Rs. 27,750 as penalty, find the number of days for which the construction work is delayed. Sol. Let the delay in construction work be for n days. Here $a = 200, d = 250 - 200 = 50, S_n =$ 27,750 . $S_n = \frac{n}{2}[2a + (n-1)d]$ .: 27,750= $\frac{n}{2}[2 \times 200 + (n-1) \cdot 50]$ 27,750= $\frac{50n}{2}[8 + (n-1)]$ $\Rightarrow \frac{27,750}{25} = n(8 + n - 1) \Rightarrow 1110 = n(n + 7) \Rightarrow 0 = n^2 + 7n - 1110 = 0 \Rightarrow n^2 + 31n - 30n - 1110 = 0 \Rightarrow n(n + 37) - 30(n + 37) = 0 \Rightarrow (n + 37) \cdot (n - 30) = 0 \Rightarrow n + 37 = 0 \text{ or } n - 30 = 0 \text{ Rejecting } n = -37, n = 30$ (.: Number of days can not be negative) . Construction and 30 (... Number of days can not be negative)...Construction work was delayed for 30 days If two tangents are drawn to a circle from an external point, then Q.32 (i) They subtend equal angles at the centre. (ii) They are equally inclined to the segment, joining the centre to that point. Solve for x: $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ : $a \neq 0, b \neq 0, x \neq 0$ . Ans = -a & -bQ.33 A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90 find the number of articles produced and the cost of each article. Ans. Articles 6,15 An iron pillar has lower part in the form of a right circular cylinder and the upper part in the form of a Q.34 right circular cone. The radius of the base of each of the cone and the cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if 1 cm<sup>3</sup> of iron weighs 7.5 grams. (Take $\pi = \frac{22}{7}$ ) Sol. Radius of base of the cylinder, (r) = 8 cm Radius of base of the cone, (r) = 8 cm Height of cylinder, (h) = 240 cm Height of cone (H) = 36 cm 8 cm Total volume of the pillar = Volume of cylinder + Volume of cone $= \pi r^2 \left( h + \frac{1}{3} H \right) = \frac{22}{7} \times 8 \times 8 \left( 240 + \frac{1}{3} (36) \right) \Rightarrow \frac{1408}{7} (240 + 12) \text{ cm}^3 \frac{1408}{7} \times 252 = 50688 \text{ cm}^3$ $= \text{pillar} = 50688 \times \frac{7.5 \text{ (gms.)}}{1000} \text{ kg} \frac{380160}{1000} = 380.16 \text{ kg}$ $\therefore$ Weight of the pillar = 50688 x

<u>IT'S CHOICE - NOT CHANCE - THAT DETERMINES YOUR DESTINY.</u>

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